

TITLE:

RAPID COOLING AND STRUCTURE OF NEUTRON STARS

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Rapid Cooling and Structure of Neutron Stars

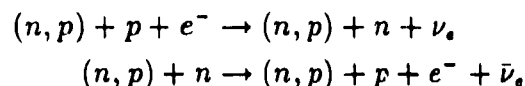
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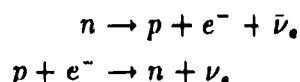
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1 DIRECT URCA NEUTRINO EMISSION

Neutrino emission in the core of a neutron star by the modified URCA process



requires the existence of a spectator nucleon to simultaneously conserve energy and momentum. Each degenerate nucleon involved in the reaction contributes a factor $T/T_F \ll 1$ to the emission rate, where T is the temperature and T_F is the Fermi temperature. Lattimer et al. (1991) have pointed out that the direct Urca process



can occur in the degenerate matter of the neutron star core if the proton/nucleon ratio is in excess of some critical value lying in the range 0.11–0.15. We have constructed a number of high density equations of state (EOSs 1–6) which satisfy this criterion; our numerical calculations make use of these for the neutron star model. Prakash et al. (1992) showed that matter with *any* proton/nucleon ratio can rapidly cool by the direct Urca process if Λ hyperons exist. Nearly all studies of supranuclear matter to date indicate that one or the other, or both, of these situations will take place.

The direct Urca emissivity is

$$\begin{aligned}\epsilon_{\text{Urca}} &= \frac{457\pi}{10080} \frac{G_F^2(1+3g_A^2)}{\hbar^{10}c^5} m_n m_p \mu_e (kT)^4 \Theta_i \\ &= 4.24 \times 10^{27} (Y_e n/n_s)^{1/3} m_n^* m_p^* T_9^4 \Theta_i \text{ erg cm}^{-3} \text{ s}^{-1}\end{aligned}$$

where $T_9 = T/10^9$ deg, $n_s = 0.16 \text{ fm}^{-3}$ is nuclear density, m_n and m_p are the effective nuclear masses, m_n^* and m_p^* are these masses divided by the nucleon mass, and the

threshold factor $\Theta_i(x) = 1$ if $x > 0$, 0 otherwise. For the dense matter equations of state we assume, the threshold factor limits direct Urca emission to densities greater than $6.9 \times 10^{14} \text{ g cm}^{-3}$. The overall coefficient is 5 orders of magnitude larger than the rate for the modified Urca process $\approx 10^{22} (Y_e n/n_s)^{1/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}$, and the direct Urca falls more slowly with T . When either the neutrons or protons, or both, are superfluid, the direct Urca rate is suppressed by the factor $\exp\left(\frac{T}{T_c^n}\right) \exp\left(\frac{T}{T_c^p}\right)$, where T_c are the critical temperatures corresponding to the superfluid gaps.

2 THERMAL EVOLUTION MODELS

We made numerical calculations of the evolution of neutron stars cooling by the direct Urca and other accelerated cooling mechanisms using the computer code developed by Van Riper (1988, 1991). This code uses a diffusion algorithm to follow both the conduction of heat and energy losses by neutrino emission inside the star; Van Riper (1991) describes the thermal conductivities, neutrino emissivities and heat capacities used. We compute the temperature distribution interior to the density $10^{10} \text{ g cm}^{-3}$ and treat the atmosphere external to this as a boundary condition using the models of Van Riper (1988). For most models, a surface magnetic field strength $B = 10^{12} \text{ G}$ was assumed. Magnetic field effects are important only at densities $< 10^{10} \text{ g cm}^{-3}$. At higher densities a field $B \leq 10^{13} \text{ G}$ makes little impact on the conductivity, heat capacity, and equation of state, and thus will not effect the cooling wave.

Accelerated core cooling, such as by the direct Urca process, results initially in a temperature inversion. After a few months, the core of matter with densities greater than nuclear has temperatures about a few $\times 10^8 \text{ deg}$, while the overlying crust is an order of magnitude hotter. Energy in the crust conducts to the core, resulting in a cooling wave which moves outward on a time given by the thermal conduction time in the crust. The surface temperature drops precipitously, as shown in Figure 1, when the cooling wave reaches the surface.

We define a cooling time t_w , indicated by filled circle in Figure 1, as the time at which the slope of the cooling curve is the steepest. Before this time, the surface temperature T_s is the same as for the standard cooling case (and depends only on the properties of the crust). After t_w , the interior of the star has become isothermal, and T_s is determined by the global balance of emission and residual thermal energy.

A numbers of models with direct Urca and other accelerated cooling mechanisms, such as the the Urca process on percolating quarks (Kiguchi and Sato, 1981), give a range of cooling times $1 \leq t_w \leq 700 \text{ years}$. The cooling time increases with the radius R of the star, as is shown in Figure 2. A power law fit to $t_w(R)$ ($\propto R^6$ is best) is rather poor. A much better power law results if we consider t_w as a function of the crust thickness $R_{\text{shell}} = R - r$ ($\rho = \rho_{\text{nuclear}}$). The exponent of the fit, 1.8–2.0, is the value expected from our analytic considerations below. The solid curves in Figure

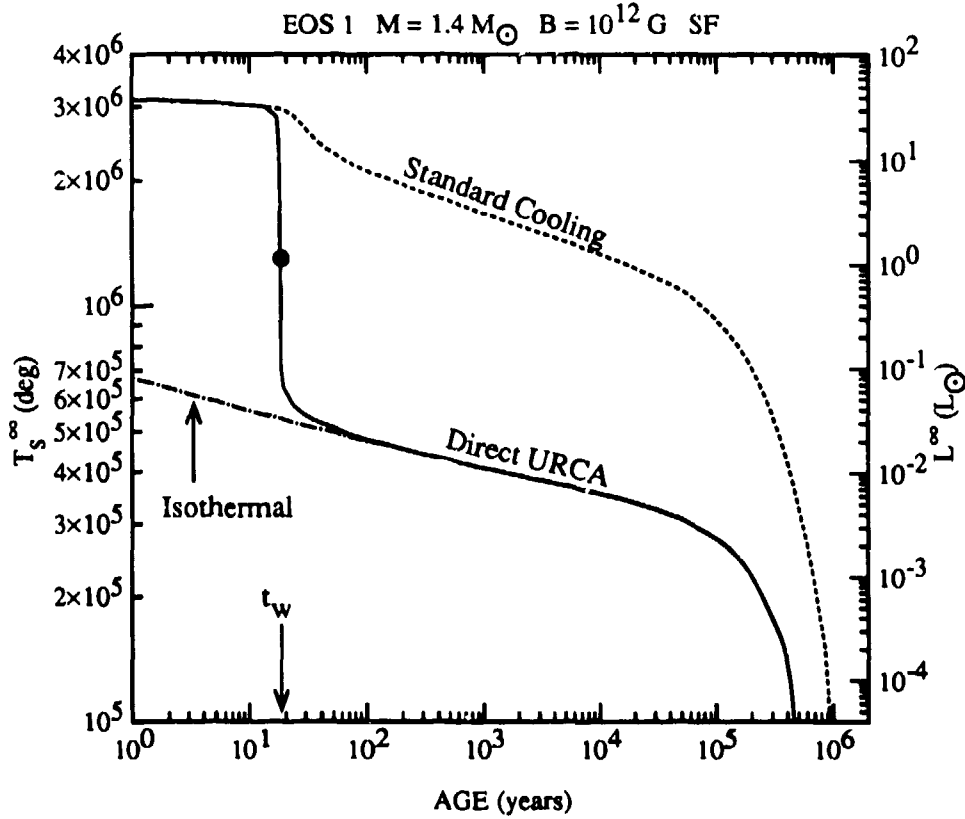


FIG. 1.— Cooling curves for a $1.4 M_{\odot}$ neutron star with a surface magnetic field strength 10^{12} G constructed with EOS 1 and including standard superfluid effects. The dashed line assumes a “standard” neutrino emissivities. The solid curve results when direct Urca emission occurs in the core. The filled circle shows our definition of the cooling time t_w . The dot-dashed curve assumes an isothermal interior and the direct Urca process; the interior of the realistic star has become isothermal when this and the solid curve coincide.

2 are a sequence of models of $M = 1.4 M_{\odot}$ stars with different high density equations of state. The dashed curve is from varying the mass with one equation of state—our EOS 2 which has compressibility of 180 MeV at nuclear density and gives a maximum neutron star mass of $1.45 M_{\odot}$. When superfluid effects are not included, the cooling time increases by a factor of 3. This is due to the lack of superfluid suppression of the neutron heat capacity C_V in the crust and the dependence $t_w \propto C_V$. The factor of 3 depends on an average in both space and time of the fraction of superfluid neutrons in the crust and the contribution of the neutrons to C_V relative to the electron contribution, which is unaffected by superfluidity.

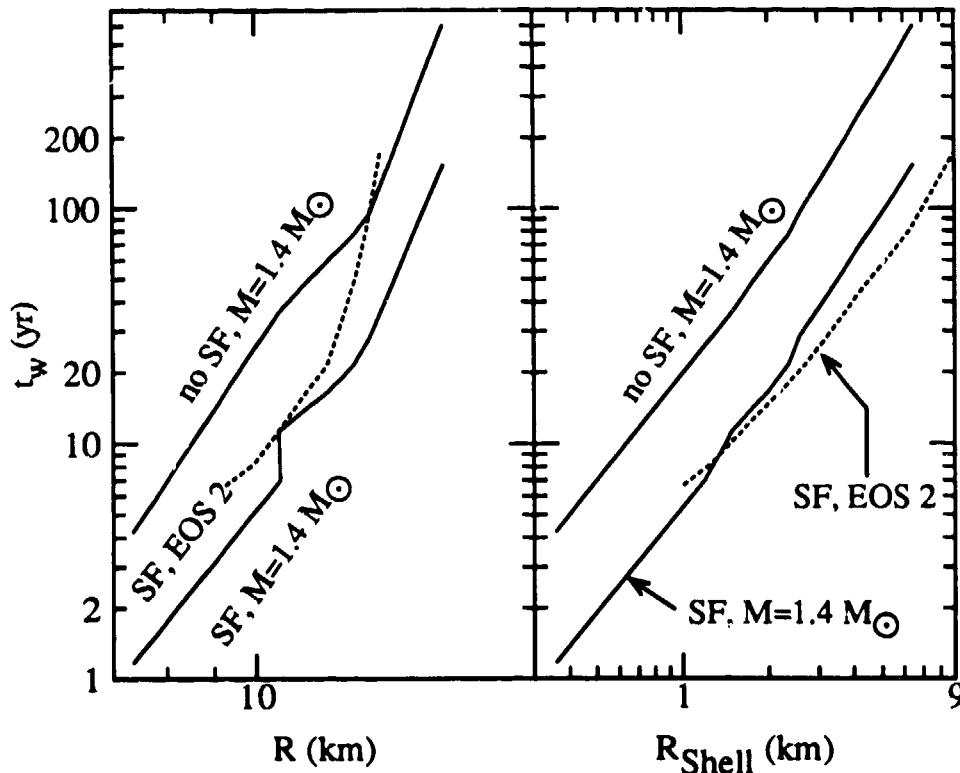


FIG. 2.— The cooling time t_w as a function of stellar radius (left panel) and the crust thickness (right panel)

3 ANALYTIC MODEL FOR DIFFUSION THROUGH THE CRUST

We have developed analytic and semi-analytic models for the cooling wave in the crust to find the dependence of the cooling time on stellar properties such as the crust thickness. We consider only conductive transport of energy in the crust; neutrino emission there takes place a time scale longer than t_w at an age of t_w . General relativistic terms are also neglected for the sake of simple equations. (These approximations are not made in the numerical calculations.) The equations of radiative transport and energy balance are

$$L = -4\pi r^2 K \frac{\partial T}{\partial r} \quad \text{and} \quad \frac{\partial L}{\partial r} = -4\pi r^2 C_v \frac{\partial T}{\partial r},$$

where L is the luminosity, r is the radial coordinate, and K is the thermal conductivity. A good fit to the latter for a large region of (T, ρ) space is $K \simeq A(\rho/\rho_o)/T$ with $A \simeq 4 \times 10^{20} \text{ erg cm}^{-1} \text{ s}^{-1}$ and $\rho_o = 2.7 \times 10^{14} \text{ g cm}^{-3}$. The specific heat can

approximated by $C_V \simeq B(\rho/\rho_o)^{1/3}T$ with $B = 1.6 \times 10^{11} \text{ erg K}^{-2} \text{ cm}^{-3}$. Combining all this gives a single equation

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(\frac{A \rho r^2}{\rho_o T} \frac{\partial T}{\partial r} \right) = B \left(\frac{\rho}{\rho_o} \right)^{1/3} T \frac{\partial T}{\partial t}$$

for conduction in the crust. Here we consider the simple case $\rho = \text{constant}$; more general solutions can be found in Lattimer et. al. (1992). We assume that the solution for T is separable in time and space, $T = T_o \psi(r) \phi(t)$, where T_o is the initial temperature at the outer edge of the crust (ie. at $t = 0$, $r = R$). The boundary conditions are taken to be $\psi(R) = 1$, $\psi'(R) = 0$, $\phi(0) = 1$, and $\psi(R_{\text{core}}) = T_{\text{core}}/T_{\text{surface}} \sim 1/30$. Defining a dimensionless depth

$$x = \frac{R - r}{R - R_{\text{core}}}$$

and shell thickness

$$q = \frac{R}{R - R_{\text{core}}},$$

we have

$$\frac{1}{\psi^2} \frac{d}{dx} \frac{1}{\psi} \frac{d\psi}{dx} + \frac{2}{\psi^3(x+q)} \frac{d\psi}{dx} = \frac{B}{A} \left(\frac{\rho}{\rho_o} \right)^{-2/3} (R - R_{\text{core}})^2 T_o^2 \phi \frac{d\phi}{dt} = -\alpha$$

where α is a separation constant. The solution for the time function is

$$\phi = \sqrt{1 - \frac{t}{\tau}}$$

where

$$\tau = \frac{B T_o^2 (R - R_{\text{core}})^2}{A \alpha (\rho/\rho_o)^{2/3}} \simeq 130 \left(\frac{T_o}{10^9 \text{ K}} \right)^2 \left(\frac{R - R_{\text{core}}}{\text{km}} \right)^2 \frac{1}{\alpha (\rho/\rho_o)^{2/3}} \text{ yrs}$$

Note the proportionality to $B \propto C_V$ and the 2nd power of the shell thickness. If we assume a thin shell, $q \ll 1$, the radial equation becomes

$$\frac{1}{\psi^2} \frac{d}{dx} \frac{1}{\psi} \frac{d\psi}{dx} = -\alpha,$$

which is solved by

$$\psi = \frac{1}{\cosh[\sqrt{\alpha} x]}$$

with

$$\alpha = [\cosh^{-1} \psi(1)^{-1}]^2 \simeq 17,$$

which has a weak dependence on the core temperature.

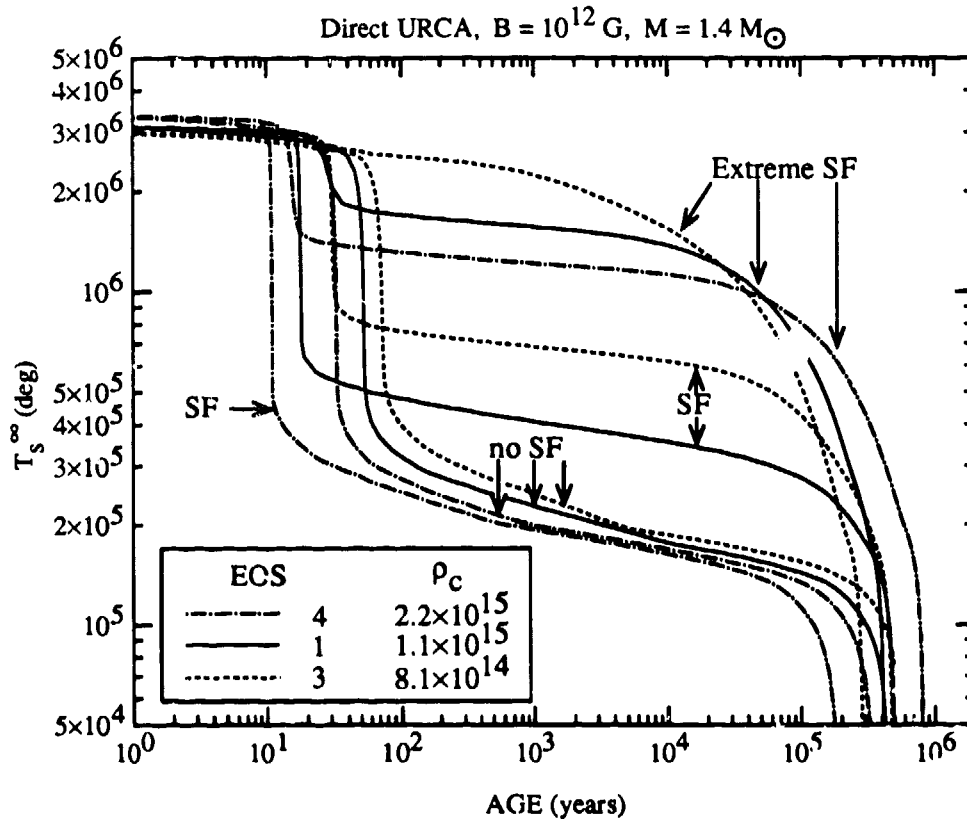


FIG. 3.— Cooling curves for $1.4M_{\odot}$ neutron stars cooling by the direct Urca process. The different line styles correspond to 3 different EOSs which result in the central densities shown. For each EOS, superfluid suppression of emissivities and heat capacities was neglected (no SF) or treated with standard superfluid parameters (SF) or with enhanced gaps above nuclear density (Extreme SF).

4 CORE SUPERFLUIDITY

The direct Urca process is suppressed by a factor $\exp\left(-\frac{T}{T_c}\right)$ for neutrons and/or protons when those nucleons are superfluid. The reduction in the cooling rate and the consequent effect on the cooling curves can be large if a large portion of the direct Urca emitting core is superfluid. The effect is much more than in the standard cooling case where crust bremsstrahlung emission, which persists in superfluid regions, is comparable in total emissivity to the modified Urca process.

Figure 3 shows cooling curves for three variations of superfluid treatment. Superfluid suppression of the emissivity and heat capacity does not occur in the “no SF” case. The “SF” model treats suppression with gaps based on realistic calculations (Takatsuka 1972, Chao, Clark, and Yang 1972). The “extreme SF” model exagger-

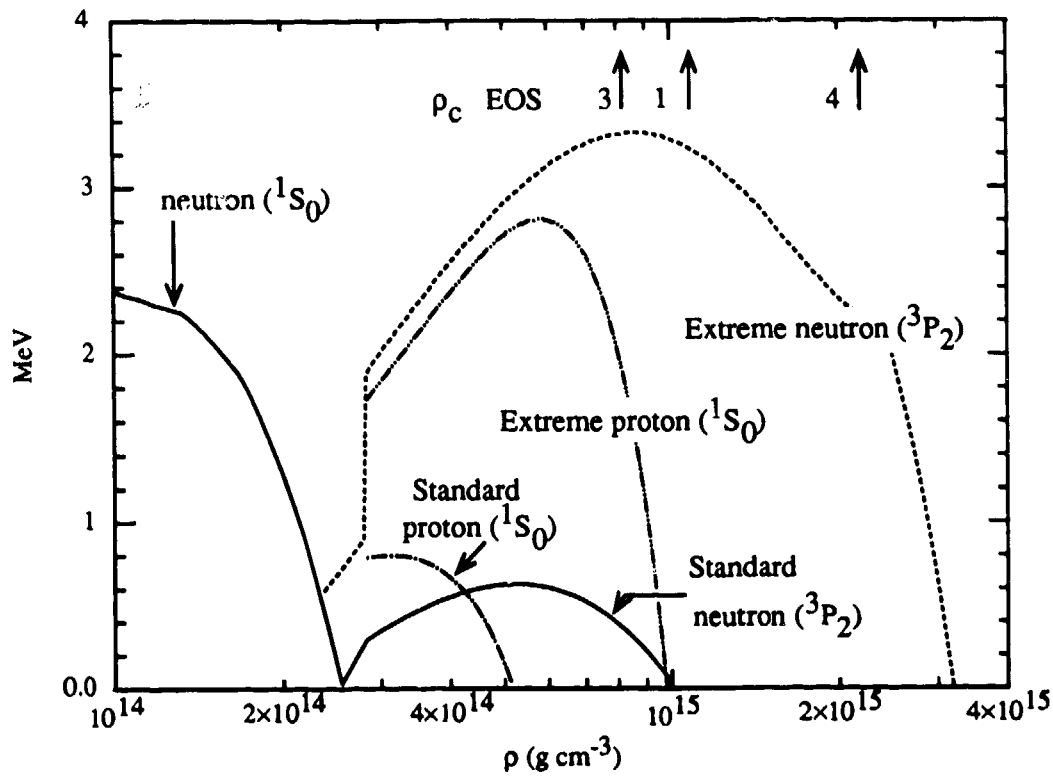


FIG. 4.—The superfluid gaps used the calculations. The same neutron 1S_0 gap is used for both the standard and extreme models. The arrows along the top show the central densities of the stars whose cooling is shown in Figure 3.

ates the neutron 3P_2 and proton 1S_0 gaps above nuclear density by increasing both their height and density range. The standard and extreme gaps are shown in Figure 4. The neutron 1S_0 gap in the crust is the same in both models. In addition to exploring the effects of uncertainties in these two gaps, the extreme model is useful in estimating the consequences of gaps at higher densities due to higher order pairings.

Direct Urca cooling was calculated for three equations of state. The stiff EOS 3, which has a maximum neutron star mass of $2.06M_\odot$, results in a $1.4M_\odot$ neutron star having a central density ρ_c just above the direct Urca threshold. A soft case and a high ρ_c is represented by EOS 4, for which the maximum mass is $1.44M_\odot$. Figure 4 shows the ρ_c of these three neutron stars. The direct Urca emitting region extends from the threshold of $6.9 \times 10^{14} \text{ g cm}^{-3}$ to the central density; the magnitude of the superfluid effect depends on the overlap of the gaps with this density range. The relative overlap decreases with increasing central density, and Figure 3 shows a large change in the cooling curve for the SF models for equations of state 3 and 1

which have a large amount of superfluid matter in the core. The central density for equation of state 4 is well above the reach of the standard neutron 3P_2 gap; the major difference between the SF and no SF models for equation of state 4 is the change in t_w due to the superfluid suppression of the heat capacity in the crust. The extreme SF gaps nearly obviate the role of the direct Urca process as an accelerated cooling mechanism.

5 CONCLUSIONS

A neutron star cooling very rapidly in the center will undergo a sharp decrease in surface temperature at the time t_w for thermal diffusion through the crust. This time depends on the square of the thickness of the crust, and is also influenced by the details of the neutron superfluidity in the crust. An observational determination of t_w will limit the radius of the star, and hence the mass and high density equation of state.

For stars cooling by the direct Urca process, the surface temperature in the epoch between the drop at t_w until an age of about 10^6 years depends strongly on the details of the superfluid gaps above nuclear density and on the central density of the star. The parameters of the high density gaps are uncertain, as is the central density. It is also possible that higher order gaps exist for both the neutron and protons; these gaps will also influence the surface temperature for $t > t_w$.

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